

Highly Excited Mesons, Linear Regge Trajectories and the Pattern of the Chiral Symmetry Realization

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Abstract

The chiral symmetry of QCD shows up in the linear Weyl–Wigner mode at short Euclidean distances or at high temperatures. On the other hand, low-lying hadronic states exhibit the nonlinear Nambu–Goldstone mode. An interesting question was raised as to whether the linear realization of the chiral symmetry is asymptotically restored for highly excited states. We address it in a number of ways. On the phenomenological side we argue that to the extent the meson Regge trajectories are observed to be linear and equidistant, the Weyl–Wigner mode is not realized.

This picture is supported by quasiclassical arguments implying that the quark spin interactions in high excitations are weak, the trajectories are linear, and there is no chiral symmetry restoration. Then we use the string/gauge duality. In the top-down Sakai–Sugimoto construction the nonlinear realization of the chiral symmetry is built in. In the bottom-up AdS/QCD construction by Erlich et al., and Karch et al. the situation is more ambiguous. However, in this approach linearity and equidistance of the Regge trajectories can be naturally implemented, with the chiral symmetry in the Nambu–Goldstone mode.

Asymptotic chiral symmetry restoration might be possible if a nonlinearity (convergence) of the Regge trajectories in an “intermediate window” of n, J , beyond the explored domain, takes place. This would signal the failure of the quasiclassical picture.

1 Introduction

Since the inception of QCD till the end of Millennium the prime interest of the QCD practitioners was the spectrum and properties of the low-lying hadronic states, such as ρ mesons, pions and nucleons. A number of methods was developed to treat such states, starting from the soft-pion technique which predates QCD by a decade, then QCD sum rules, lattice calculations and so on. Little attention was paid to highly excited states. The reason is obvious: the decay widths of the excited states grow with the excitation number, so that they overlap and collectivize themselves, and could be treated as continuum.

In the Regge theory which dominated high energy theory before QCD, highly excited states played an important role in phenomenological analyses since they determine the daughter Regge trajectories. The Regge theory gave rise to dual resonance models which eventually grew into string theory. Ironically, string theory that emerged from the dual resonance models shortly after became “string theory for nonhadrons,” and was elevated to the status of “theory of everything” in the 1980s and early ’90s. With this promotion the previous interest to excited hadronic states faded away. At the same time, in QCD highly excited states were treated as belonging to the realm of asymptotic freedom which inevitably qualified them as “dynamically uninteresting objects.”

This attitude changed in recent years with the advent of string–gauge duality methods, based on the ’t Hooft limit [1] with the number of colors $N_c \rightarrow \infty$ while $g^2 N_c$ is kept fixed. In this limit the meson decay widths tend to zero, so that individual highly excited mesons become well-defined.¹

The string–gauge duality-based ideas predict a certain pattern for excited resonances. On the other hand, significant amount of data regarding excited mesonic resonances was accumulated. These data shown in Fig.1 exhibit a high degree of degeneracy.² In classical strings the Regge trajectories are linear and equidistant implying the spectral degeneracy. The mode of coexistence of the chiral symmetry with the Regge trajectories is a challenging theoretical issue.

Generically, the chiral symmetry could be realized linearly, i.e. in the Wigner–Weyl mode, when chiral multiplets contain degenerate states of the opposite parity, or non-linearly, in the Nambu–Goldstone mode, in which the action of symmetry

¹ Baryons, if treated in the standard ’t Hooft procedure, defy this rule; their decay widths, generally speaking, do not vanish in the limit $N_c \rightarrow \infty$, also their masses grow as N_c . However, the $N_c \rightarrow \infty$ limit exists for the mass differences, and experiments show that rather high excitations of nucleons and other baryons can be identified using the existing data.

² We should warn the reader that there is no consensus among experts with regards to some resonances at higher levels and some selection criteria for $q\bar{q}$ states, see the figure caption.

The above dichotomy — restoration vs. nonrestoration — is in the focus of the present work. We first explain that, purely theoretically, the Nambu–Goldstone mode of the chiral symmetry implementation for the low-lying states could coexist with the linear realization for high excitations. Which of the two alternatives takes place in actuality is decided by dynamics.

In the first part of our paper we focus on high radial excitations in $q\bar{q}$ mesons where $q = u$ or d . We argue that the observed approximate linearity in the $\{M^2, J\}$ plane and equidistance of the $q\bar{q}$ -meson Regge trajectories — to the extent and in the domain they are observed — imply nonrestoration of the linear chiral symmetry.

The genuine restoration means that the mass difference between the would-be chiral partners of the opposite parities is much less than, say, the gap to the next radial excitation. However, for linear trajectories they both scale as $1/M$. Under the circumstances, the very notion of the “chiral partners” becomes meaningless; rather we deal with an “extended promiscuous family.” A large number of states of positive and negative parity are connected with each other by axial transitions. This is typical of the Nambu–Goldstone, rather than Wigner–Weyl mode: excited mesons are not decoupled from the pion and the axial transitions between mesons can have arbitrary strengths.

Our conclusion is in obvious contradiction with the interpretation suggested in early works [5–7] in which the $1/M$ fall-off of the mass splittings between the chiral partners was considered to be a signal of the asymptotic χ SR. Theoretical basis for some of these works was provided by a straightforward four-dimensional extension [11, 13, 14] of the two-dimensional ’t Hooft model [15, 16].

This two-dimensional model is fully understood. For massless quark it has $U(1)_L \times U(1)_R$ chiral symmetry. This symmetry is spontaneously broken by the quark condensate, a massless Goldstone meson ensues. The fact that the chiral symmetry is not restored in high radial excitations (needless to say, there are no orbital excitations in two dimensions) is clearly seen from the Goldberger–Treiman relation

$$g_{\pi+-} = f_\pi^{-1} g_A (M_+^2 - M_-^2),$$

where $g_{\pi+-}$ is the pion coupling to $|\pm\rangle$ mesons of opposite parity, with masses M_\pm . The excitation mass spectrum is known to be equidistant in M^2 ; for the nearest neighbors $M_+^2 - M_-^2$ is independent of the excitation number. While the pion decouples in the $N_c \rightarrow \infty$ limit, the coupling $g_{\pi+-}$ does not fall off with the excitation number.

Unlike the two-dimensional ’t Hooft model, the four-dimensional QCD dynamics is not fully understood. Therefore, generally speaking, the observed pattern of the Regge trajectories could change outside the measured domain of $\{M^2, J\}$. In particular, the chiral partners could approach each other at large J . This would certainly

imply a strong nonlinearity of the Regge trajectories at the right edge of Fig. 1 and beyond. Although data give no hint of such a behavior, logically it is not ruled out.

What does theory say on this issue?

In the framework of holographic string/gauge duality the most developed construction was suggested by Sakai and Sugimoto [17]. Albeit this construction does not reproduce the linearity of the Regge trajectories, the Nambu–Goldstone mode of the chiral symmetry realization is built-in.

On the other hand, the bottom-up AdS/QCD approach of Erlich et al. [18] and Karch et al. [19] does not lead to a unique answer. Although in Ref. [19] the authors obtained asymptotically linear realization it was at a price of nonlinearity of the Regge trajectories (as a function of n). Moreover, within the same approach one can change a certain assumption (see, e.g. [20, 21]) ensuring the linearity of the trajectories and keeping the Nambu–Goldstone mode for high excitations.

While the analysis based on the string/gauge duality is still open for interpretations, arguments based on quasiclassical considerations seem to be much more definite. They imply that there is no χ SR at high n , J . If a convergence of trajectories at modestly large values of J was found (signaling the beginning of χ SR in the large- J limit), this would defy the quasiclassical picture.

A comment on baryons is in order here. There is no clean formulation of the problem for arbitrarily large n and J since, as was mentioned, the large- N_c limit does not help for baryons: starting from a certain value of the excitation number they are expected to overlap and become unidentifiable. However, given the fact that excitations of N and Δ with intermediate values of n and J are observed, one can pose the question of χ SR with regards to these “intermediate” excitations. We discuss this question in the second part of our paper.

We also discuss a relation between the chiral symmetry mode and relevant distances. At short Euclidean separations the chiral symmetry is restored, which is obvious from the operator product expansion (OPE). However, Euclidean considerations are not sensitive enough to the relative positions of individual chiral partners. Looking from the Minkowski side, we observe the growth of characteristic distances with energy: the meson size grows linearly with its mass. Note that the chiral symmetry restoration at high temperatures is similar to that at short Euclidean distances. In both cases we average over a large number of resonances.

This paper is organized as follows. In Sect. 2 we discuss how the nonlinear and asymptotically linear realizations could coexist, in principle, at low and high energies, respectively. In Sect. 3 we classify the $\bar{q}q$ meson Regge trajectories assuming χ SR. In Sect. 4 we explain that linearity of the $\bar{q}q$ meson trajectories (in the absence of dislocations) implies the Nambu–Goldstone mode. Section 5 presents a quasiclassical

picture of high excitations of $q\bar{q}$ mesons which illustrates our conclusions. In Sect. 6 we consider the meson decay widths and estimate g_A . Section 7 is devoted to the issue of implementation of the chiral symmetry within the string/gauge duality formalism. In Sect. 8 we discuss chiral symmetry from the Euclidean side. Section 9 presents a brief discussion of the baryon Regge trajectories. Finally, Sect. 10 summarizes our conclusions. In Appendix A we review representations of the linear chiral symmetry. In Appendix B we discuss generalized Goldberger–Treiman relations.

2 Coexistence of the Nambu–Goldstone and linear realizations of the chiral symmetry

The QCD Lagrangian with N_f massless quarks possesses $U(N_f)_L \times U(N_f)_R$ chiral symmetry (the axial $U(1)$ is anomalous at the quantum level). The linear representations of this symmetry were studied in the literature in detail; we review them in Appendix A.

We know for certain that this chiral symmetry is spontaneously broken which implies the existence of the massless Goldstone bosons, pions. A manifestation of this breaking is non-degeneracy of the chiral partners, say, ρ and a_1 mesons. Thus, at low energies the chiral symmetry is implemented in the Nambu–Goldstone mode.

Then, how could this symmetry be asymptotically restored in high excitations?

For illustrative purpose consider a simple case of $U(1)_L \times U(1)_R$ chiral symmetry,³ having in mind the its generalization to $U(2)_L \times U(2)_R$, for two massless flavors is quite straightforward. In the linear realization the symmetry generators V and A act as

$$V|\pm\rangle = |\pm\rangle, \quad A|\pm\rangle = |\mp\rangle, \quad (1)$$

where $|\pm\rangle$ are opposite parity states whose masses are degenerate, $M_+ = M_-$.

The matrix element of the axial current a^μ between any opposite-parity states generically has the following form:

$$\langle +|a^\mu|-\rangle = g(q^2)(p_+ + p_-)_\nu \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) = g(q^2) \left[(p_+ + p_-)^\mu - q^\mu \frac{M_+^2 - M_-^2}{q^2} \right], \quad (2)$$

where $q = p_+ - p_-$ is the momentum transfer and we assume, for simplicity, that $|\pm\rangle$ are spin-0 particles (generalization to higher spins is straightforward). The specific

³ Thus, we neglect dynamical breaking of the $U(1)_A$ symmetry through anomaly. The effect of breaking vanishes at $N_c = \infty$. At finite N_c this effect is suppressed by assumption of asymptotic χ SR.

form above is dictated by conservation of the axial current. The pole at $q^2 = 0$ reflects propagation of the massless pion.

In the linear realization, when $|+\rangle$ and $|-\rangle$ are partners, their masses are degenerate,

$$M_+^2 = M_-^2 \quad (3)$$

and the axial coupling

$$g_A = g(0) = 1 \quad (4)$$

to ensure that the axial generator $A = \int d^3x a^0$ acts as in Eq. (1). As seen from Eq. (2), the degeneracy (3) implies pion decoupling.

When the chiral symmetry is spontaneously broken the masses are not equal, $M_+^2 \neq M_-^2$, and the coupling to pion does not vanish, it is given by the generalized Goldberger–Treiman relation which follows from Eq. (2) (see also Appendix B),

$$g_{\pi+-} = f_\pi^{-1} g_A (M_+^2 - M_-^2). \quad (5)$$

There are no constraints on the axial coupling g_A because the matrix element of the axial generator A vanishes, $\langle -|A|+\rangle = 0$, as seen from Eq. (2) at $\vec{q} = 0$. Indeed in the spontaneously broken phase the action of A just adds a soft pion to the $|+\rangle$ state instead of converting it into $|-\rangle$.

How is it possible to interpolate between the nonlinear realization of the chiral symmetry for low-lying states and restoration of the linear realization for highly excited states? Imagine that the mass difference $\Delta M_\pm = M_+ - M_-$ is small for high excitations, i.e., much smaller than a scale M_h at which the form factor $g(q^2)$ changes. Then the matrix element (2) for a^0 in the rest frame of $|+\rangle$ takes the form

$$\langle -|a^0|+\rangle = g(q^2) 2M_- \frac{\vec{q}^2}{\vec{q}^2 - (\Delta M_\pm)^2}, \quad (6)$$

where we imply $|\vec{q}| \ll M_\pm$ for the spatial momentum transfer and neglected $\Delta M/M$ corrections. We observe the above-mentioned vanishing of this matrix element at $\vec{q} = 0$.

For small ΔM_\pm there could be a range of \vec{q}

$$\Delta M_\pm \ll |\vec{q}| \ll M_h, \quad (7)$$

where

$$\langle -|a^0|+\rangle = 2M_- g(0) = 2M_- g_A. \quad (8)$$

This can be interpreted as the integral over a^0 over a finite spatial range $R \sim 1/|\vec{q}|$, in contrast with the $\vec{q} = 0$ case when integration for the axial charge includes all

space. Moreover, this implies that for high excitations chiral symmetry is restored in its linear form at the scales (7) and $g_A = 1$.

For the scenario above to work the condition $\Delta M_{\pm} \ll M_h$ must be satisfied. The parameter M_h is related to the splitting between the neighboring states of the same parity $M_{n+1} - M_n$ (for extended objects of size R it is the same as $1/R$). Examining Fig. 1 one can see that M_h defined above is of the same order as ΔM_{\pm} . Thus, the window (7) does not exist.

This is particularly clear for the states on the leading Regge trajectories, i.e. the minimal mass states for the given spin. There are no chiral partners on the leading trajectory. The splittings between the would-be chiral partners is of the same order as the splittings between the same parity states. It shows that a simple pattern of the chiral symmetry restoration is not realized in nature, at least in the explored range of excitations on the $\{M^2, J\}$ plane.

A different scenario could be realized if

$$\Delta M_{\pm}^2 = M_+^2 - M_-^2 \quad (9)$$

tended to zero for high excitations. Then, at sufficiently high n , the splittings between the states in the would-be chiral pairs would become much smaller than the splittings between the states of the same parity (in the $U(2)_L \times U(2)_R$ case, the splittings inside the would-be linear chiral multiplets, see Appendix A, would become much smaller than the splittings between distinct multiplets). Then the pion coupling becomes weak and can be neglected. Simultaneously, g_A 's inside the multiplet tend to 1, while g_A 's for transitions connecting distinct multiplets tend to zero. In this case the states from the given chiral multiplet can be considered in isolation from the rest. This pattern of chiral symmetry realization could be referred to as asymptotic restoration.

A priori the rate of vanishing of ΔM_{\pm}^2 in this scenario can be arbitrary. It depends on underlying dynamics. A natural scaling law is

$$\Delta M_{\pm}^2 \propto M^{-2} \propto (n^{-1}, J^{-1}) \quad \text{or} \quad \Delta M_{\pm} \propto M^{-3} \propto (n^{-3/2}, J^{-3/2}). \quad (10)$$

The above expressions can serve as a benchmark scaling law. Note that in the case of constant ΔM_{\pm}^2 we get ΔM_{\pm} still decreasing for high excitations as $1/\sqrt{n}$. In the literature this was often viewed as sufficient for restoration of linear chiral symmetry. As was argued above, this is not the case. The splittings of the chiral partners must be much smaller than other splittings to produce the genuine restoration. We discuss this in more detail in the subsequent sections.

3 Quark-antiquark meson Regge trajectories

Let us focus on $\bar{q}q$ mesons. Such states are characterized by the total quark spin $S = 0, 1$ (singlet or triplet) and total angular momentum J . In nonrelativistic models one can also define the orbital momentum $L = J$ for singlets and $L = J \pm 1, J$ for triplets. In the relativistic case, while L itself is not a good quantum number, spatial and charge parities (for neutral mesons),

$$P = (-1)^{L+1}, \quad C = (-1)^{L+S}, \quad (11)$$

are well defined and conserved. Then, $L \bmod 2$ and $S \bmod 2$ are good quantum numbers. Since S can be only 0 or 1 this means that S is a good quantum number too. Correspondingly, there are spin singlet Regge trajectories for which $P = -C = (-1)^{J+1}$, and spin triplet ones with $P = C = (-1)^J$ and $P = C = (-1)^{J-1}$. The parity flip (at given J) implies a change in L by one unit.

Thus, the only new relativistic feature is a mixing between $L = J \pm 1$ states.⁴ There are two types of spin triplet Regge trajectories associated with two different combinations of $L = J + 1$ and $L = J - 1$. Assuming χ SR allows us to distinguish these combinations by the $U(1)_A$ quantum numbers.

Suppose that the chiral $U(N_f)_L \times U(N_f)_R$ would be realized linearly. How this would be reflected in the above classification? In this case, as it is discussed in Appendix A, it is convenient to classify the quark-antiquark pairs by their chirality content:

$$\bar{q}_L q_L, \quad \bar{q}_R q_R, \quad \bar{q}_R q_L, \quad \text{and} \quad \bar{q}_L q_R.$$

In fact, this is in one-to-one correspondence with the conserved $U(1)_A$ charge which is $\{0, 0, 2, -2\}$ for the above pairs. The first two pairs are $U(1)_A$ neutral while the last two are charged. This quantum number distinguishes two types of the Regge trajectories for $S = 1$ mentioned above.

To illustrate the point let us consider some examples of interpolating quark operators, see Appendix A. For instance, the ρ meson can be represented by two such operators:

$$\bar{q} \gamma^\mu \vec{\tau} q, \quad \partial_\nu [\bar{q} \sigma^{\mu\nu} \vec{\tau} q] \propto \bar{q} \vec{\tau} \overleftrightarrow{D}^\mu q. \quad (12)$$

The first one is $U(1)_A$ neutral, it contains a certain combination of S and D waves. The second is $U(1)_A$ charged and pure D wave. Correspondingly we have two distinct states with the ρ meson quantum numbers in the linear realization. What are their chiral partners? For the $U(1)_A$ neutral ρ meson it is the a_1 axial meson produced

⁴ The importance of mixing was recently emphasized in [22].

by $\bar{q}\gamma^\mu\gamma_5\vec{\tau}q$. The partner to the $U(1)_A$ charged ρ meson is h_1 associated with the operator $\bar{q}\gamma_5\overleftrightarrow{D}^\mu q$ which is singlet with respect to both, isospin and spin. The same $U(2)_L \times U(2)_R$ multiplet contains also the isotriplet axial b_1 together with isosinglet vector ω . All these 8 mesons are $U(1)_A$ charged. The chiral $(1, 0) + (0, 1)$ multiplet in the $U(1)_A$ neutral sector contains 6 mesons. Besides one should also add two isosinglets, ω_1 and f_1 , which are singlets of $U(2)_L \times U(2)_R$. They are expected to be degenerate with isotriplets for dynamical reasons. Thus, in both, $U(1)_A$ neutral and charged sectors we have 8 mesonic states. The same is valid for their Regge recurrences.

The $U(1)_A$ neutral and charged states do not mix in the linear realization.⁵ In each sector we have 8 mesons. In the $U(1)_A$ neutral sector they all have $S = 1$. The mixing of $L = J \pm 1$ is fixed by $U(1)_A$ neutrality. In the $U(1)_A$ charged sector only $L = J + 1$ is realized (of course, $L = J$ for spin singlets in the multiplet).

If we look at the spectrum of the known mesons, see Fig. 1, we do not see the abundance of the states expected in the linear realization.

4 Chiral symmetry vs. linear Regge trajectories

In the Regge picture with the linear trajectories the $\bar{q}q$ -meson resonances lie equidistantly on straight lines $M^2(J) = (J - J_0)/\alpha'$ in the plane $\{M^2, J\}$. Distinct trajectories differ only by the intercept values J_0 . Moreover, as seen from Fig. 1, on the leading trajectory, i.e., the one with no radial excitations, there is no parity degeneracy in M^2 . For, instance, ρ lies on the leading trajectory, while its “partner” a_1 lies on the first daughter trajectory.

In general, for ρ and a_1 trajectories, M^2 is not degenerate,

$$\Delta M_\pm^2 = M_+^2 - M_-^2 = \Delta J_0/\alpha' \sim \Lambda^2, \quad (13)$$

i.e. ΔM_\pm^2 does not fall off for higher- J excitations. This mass difference is of the same order as the gap between neighboring states which are not chiral partners, i.e. for neighboring radial excitations,

$$\Delta M_\pm^2 \sim M^2(\rho_{n+1}) - M^2(\rho_n), \quad (14)$$

where the subscript \pm labels the chiral partners while n refers to radial excitations.

Of course, Fig. 1 covers only a limited range of J and n . *A priori* one cannot rule out that the pattern visible in this figure changes at still higher values of J or

⁵ Although the mixing is not suppressed if chiral symmetry is spontaneously broken the number of states (Regge trajectories) remains the same.

n . Moreover, some of the data presented at level 5 are considered with reservations by experts. Still, given a rather broad range of J and n covered, it seems natural to think that this pattern will continue outside this range. Let us assume that:

(i) The (quark-antiquark) meson leading and daughter trajectories are linear and parallel to each other, at least to the extent shown in Fig. 1 (it is natural to think that at larger values of J and n the degree of linearity is even higher);

(ii) These trajectories are equidistant and there are no dislocations, i.e. each meson presented on the leading trajectory has radial excitations at every level;

(iii) The pattern of no parity-degenerate states on the leading trajectory clearly visible in Fig. 1 continues outside the range of J presented in this figure.

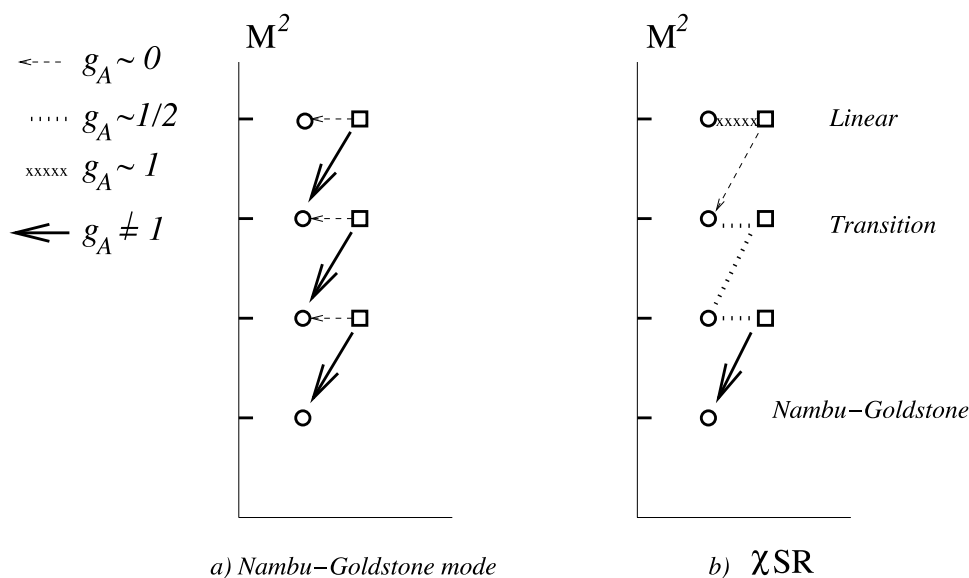


Figure 2: Two scenarios of the chiral symmetry realization for high excitations. Open circles denote ρ and its excitations, open squares a_1 and its excitations. Arrows show the values of the axial constant g_A for the corresponding transitions.

Accepting these assumptions one cannot obtain χ SR.

Let us start from the ρ meson. Its chiral partner is a_1 , the $\pi\rho a_1$ coupling is not small, and the chiral symmetry in this system is implemented in the nonlinear mode. Moreover, there are reasons to believe that $\pi\rho a'_1$ coupling is not small either and so are couplings between other neighboring excitations. If so, a few ρ mesons and a few a_1 mesons are all entangled in a network of chiral transitions with the pion emission. Clear-cut representations of $U(2)_L \times U(2)_R$ isolated from the rest of the spectrum have no physical grounds for existence. Accepting assumption (i), (ii) and (iii) above

we must say that the same statement refers to high excitations as well. The value of ΔM_{\pm}^2 cannot continuously tend to zero at large n , while a discontinuous jump is forbidden by assumption (ii). No asymptotic restoration of the chiral symmetry takes place.

This statement looks counter-intuitive. We got used to the fact that at high energies spontaneously broken symmetries are restored, the vacuum structure (i.e. a non-vanishing chirally-noninvariant quark condensate) becomes unimportant. Equations (13) and (14) imply that $\Delta M_{\pm}^2/M^2 \sim 1/n$. Although it is sufficient for symmetry restoration of inclusive quantities (such as scalar versus pseudoscalar correlators) this degree of fall-off is insufficient for χ SR. Note that the inclusive quantities correspond to measurements in the Euclidean domain, see Sect. 8.

A reservation is in order here. There is a logical possibility of a more subtle “dislocation” — a dislocation not in the spectrum but in the values of g_A ’s. This possibility is depicted in Fig. 2. As an example, this figure displays ρ , a_1 and their excitations. Assume that for the lowest-lying states, ρ and a_1 and their close neighbors, there is a set of nonvanishing g_A ’s for a number of positive-negative parity amplitudes connecting various levels. The chiral symmetry is implemented in the Nambu–Goldstone mode. For higher excitations this pattern can either continue indefinitely (see Fig. 2*a*), or, after a transition domain where g_A ’s are reshuffled (Fig. 2*b*), be replaced by a “monogamous” behavior, with $g_A = 1$ for the opposite parity states from one and the same level and $g_A = 0$ for the opposite parity states from different levels. In this scenario with increasing J the transition domain must shift to higher levels. We are aware of no dynamical scheme that could yield such a behavior.

The growth of characteristic distances with the meson mass is a natural feature in quasiclassical string picture of hadrons. In this picture, to be discussed in the next section, high excitations correspond to extended objects with growing sizes, while the Regge trajectories are linear.

5 Quasiclassical Picture

It is not clear to which extent one can quantify the quasiclassical picture. Nevertheless it seems to be instructive and it helps us introduce the notion of a pulsating QCD string and reveal the roots of a broad degeneracy in the meson spectrum.

Let us consider excited mesons which could be produced in e^+e^- annihilation into u and d quarks (ρ ’s and ω ’s). Quarks injected into the vacuum by a virtual photon at energy E travel as free objects, flying back-to-back, during the time interval $\tau_* \sim E/\Lambda^2$. During this time a string of length $\ell_* \sim E/\Lambda^2$ develops between the endpoint

quark and antiquark, which eventually absorbs the quark kinetic energy. At distance ℓ_* quarks loose their kinetic energy, become nonrelativistic (that's where χ SB is crucial), turn around, move in the opposite direction, “head-on,” and eventually stretch the string of the same length with the positions of the endpoint q and \bar{q} interchanged. Then the cycle repeats. In the limit $N_c \rightarrow \infty$ the string cannot be broken. Nor can it shake off a part of its energy in the form of glueball emission. Near each $\bar{q}q$ turning point all the energy E of the system resides in the stretched string. The endpoint quarks are slow, and their chirality can (and must) be flipped. In this way there emerges a pulsating system of q and \bar{q} connected by a long and energetic QCD string. The string can also rotate; its length grows with angular momentum. Quasiclassically, the string angular momentum and the quark spins decouple. (Empiric evidence of feebleness of spin interactions in high excitations was discussed long ago, see e.g. [23]; for a more recent consideration see [24].)

A snapshot of such highly excited meson in the limit $N_c \rightarrow \infty$ is given in Fig. 3. The quark and antiquark are attached to an unbreakable string with the tension $\sigma \sim \Lambda^2$. (In what follows we will omit inessential numerical constants and assume that the only mass dimension is provided by $\Lambda_{\text{QCD}} \equiv \Lambda$. The quark mass terms in the Lagrangian are set to zero, i.e. we will deal with the chiral limit. In this convention the string tension is Λ^2 , while the ρ -meson mass is Λ .)

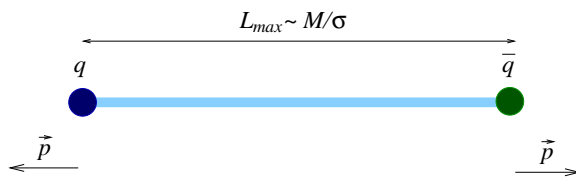


Figure 3: The quark and antiquark inside a highly excited meson, viewed quasiclassically, “oscillate” being attached to the end-points of the string that does not break at $N_c \rightarrow \infty$.

The mass of a high radial excitation of the meson state (say, ρ_n) can be determined from a quasiclassical quantization condition. The mass M_n can be presented as

$$M_n = 2p + \sigma r. \quad (15)$$

The quarks create a flux tube of the chromoelectric field with the maximal length

$$\ell_* = \frac{M_n}{\sigma}. \quad (16)$$

The quasiclassical quantization condition implies

$$\int_0^{\ell_*} p(r) dr = \pi n, \quad n = 1, 2, \dots \quad (17)$$

with $p(r) = (M_n - \sigma r)/2$. Then we immediately arrive at

$$M_n^2 = 4\pi\sigma n \sim \Lambda^2 n. \quad (18)$$

Let us parenthetically note that the asymptotically linear n dependence of M_n^2 was analytically obtained in the two-dimensional 't Hooft model [15, 16] where linear confinement is built in. In this case the next-to-leading correction is logarithmic in n .

A similar quasiclassical estimate for a spinning string implies linearity of M^2 in the angular momentum L . In fact, if both n_r and $L \neq 0$, Eq. (18) stays valid with the substitution

$$n \rightarrow n_r + L + 1. \quad (19)$$

It is important that for high excitations the length of the chromoelectric flux tube ℓ connecting a (massless) quark q with an antiquark \bar{q} is large. It scales as $\ell \sim M$.

This quasiclassical string picture above refers to the spinless constituents (of the opposite parity) at the endpoints. It should be supplemented by the endpoint quark spins. For long strings we neglect spin interaction. There are good reasons to believe that in this case spin interactions are weak. A theoretical argument is that since (chromo)magnetic charges are supposed to be condensed in the QCD vacuum the magnetic interactions are screened. A phenomenological argument is based on the pattern of degeneracy in the observed meson spectrum [23]. The spin independence means that the spectrum is the same as for spinless quarks but the multiplicity is, of course, four times larger.

Consider, for instance, the a_1 meson, $J^{PC} = 1^{++}$. It has $L = 1$ and $S = 1$. The spin independence implies that it is degenerate with the $S = 0$ state b_1 . We can show now inconsistency of the presented semiclassical picture with the linear realization of the chiral symmetry.

Let us assume χ SR. In this case the spin degrees of freedom are untangled through the $U(1)_A$ classification, as was discussed in Sect. 3. For parity partners in the chiral multiplets L is shifted by one unit. This was also mentioned in Sect. 3. If the values of n_r are the same then $M_+^2 - M_-^2 = 4\pi\sigma$. Of course, shifting n_r simultaneously by one unit we could achieve the degeneracy. However, these states would not be chiral partners because their radial “wave functions” are different and, correspondingly, $g_A \neq 1$.

We pause here to make an important remark. *Degeneracy of the opposite-parity states lying on the daughter trajectories does not automatically imply chiral symmetry restoration.* For instance, ρ' and a_1 belong to one and the same level of the first daughter, but it is highly unlikely that $g_A = 1$ in the $\rho' \rightarrow a_1$ transition. Hence,

ρ' and a_1 do not form a (part of a) linear representation of the chiral symmetry. Looking only at the mass spectrum it is easy to misinterpret such degeneracy as χ SR. Note, that the small values of the spectral overlap parameter, defined as the ratio of ΔM_{\pm} to the mass gap between the radial excitations, reported in [11] are due to such choice of the would-be chiral partners. For a related discussion of the spectral overlap see Ref. [25].

Equations (18) and (19), taken at their face value, lead to two crucial consequences. First, they predict the absence of degeneracy on the leading trajectory. Indeed, for given J the lowest M^2 state is obtained by setting $n_r = 0$ and $L = J - 1$. Equations (18) and (19) also yield linear equidistant trajectories with degeneracies on the daughter trajectories. For the same value of J the states on the first daughter trajectory can be obtained by setting $n_r = 0$ and $L = J$ or $n_r = 1$ and $L = J - 1$ (in both cases n in Eq. (19) is J). Needless to say, similar pattern extends to higher daughters.

The very same behavior — that M^2 depends on the combination $n_r + L$ — was observed in the string–gauge duality analysis of long strings in a certain approximation [26]. In this formalism, the string endpoint spins are implemented by imposing specific boundary conditions, either of the NS or R type [27]. This implies that in the approximation of Ref. [28] mesons built of (hypothetical massless) scalar quarks are degenerate with those built of the spinor quarks, which entail the quark spin orientation independence of the meson mass.

It is in order to mention here possible corrections to the above string degeneracy. If we assume that for large n_r, L

$$M \propto \sqrt{n_r + L + 1} [1 + O(\max(1/n, 1/L))] \quad (20)$$

then deviations from the string degeneracy take the form

$$\delta M^2 = O(\max(1/n, 1/L)). \quad (21)$$

This is a very interesting question which calls for further discussion.

6 Decay widths in semiclassical approximation

Now let us apply a similar quasiclassical consideration to the decay widths of high radial excitations [29,30], see also [31]. The decay probability (per unit time) is determined, to order $1/N_c$, by the probability of producing an extra quark-antiquark pair. Since the pair creation can happen anywhere inside the flux tube, the probability

must be proportional to ℓ . As a result [29–31] one gets⁶

$$\Gamma_n \sim \frac{1}{N_c} \ell_* \Lambda^2 = \frac{\beta}{N_c} M_n, \quad (22)$$

where β is a dimensionless coefficient of order 1 independent of N_c and n .

Thus, the width of the n -th excited state is proportional to its mass which, in turn, is proportional to \sqrt{n} . The square root formula for Γ_n was numerically confirmed [32] in the 't Hooft model. It is curious that both, in actual QCD and in two dimensions, $\beta \sim 0.5$.⁷

Equation (22) was recently obtained in the framework of string–gauge duality in Ref. [34] which treats only the case $L \neq 0$, $n_r = 0$. The authors calculated a subleading correction to Eq. (22) reflecting a deviation in the linear relation $\ell_* \sim M$. Inclusion of this correction improves the fit in the low-energy domain.

The result (22) can be easily understood if one takes into account the fact that the number of open typical decay channels $\sim n$, while each individual typical two-particle decay width is $\sim N_c^{-1}(\Lambda^2/M_n)$.

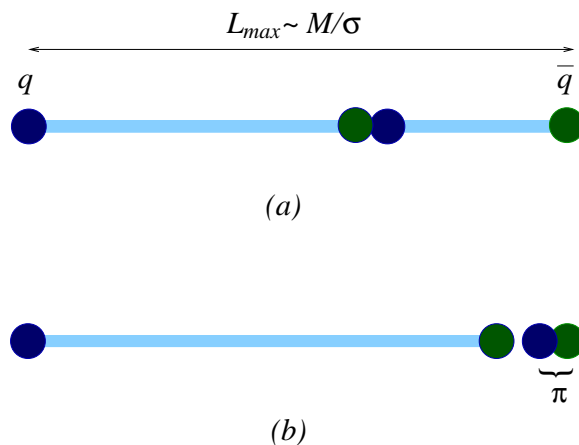


Figure 4: (a) A typical decay through pair creation producing two excited mesons in the final state; (b) A rare event with the string breaking at the end which can lead to pion emission.

If we are interested in the transition of the type $A \rightarrow B + \pi$ the estimate of the decay width drastically changes. Indeed, the pion channel is exceptional rather than

⁶ This equation clearly demonstrates that the limits $N_c \rightarrow \infty$ and $n \rightarrow \infty$ are not commutative, a rather obvious fact. We must first send N_c to infinity, and only then can we consider high excitations.

⁷ Note, however, that the fit in [33] is not consistent with (22), possibly due to contamination of the data set by non- $q\bar{q}$ mesons.

typical (see Fig. 4); the pion can be produced only if the quark-antiquark pair breaks the string of Fig. 3 close to one of the endpoints (within distance $\sim \Lambda^{-1}$), rather than in the middle of the string. Then the corresponding amplitude is

$$\propto |\vec{p}_\pi|/\sqrt{N_c} \sim \Delta M^2 g_A/f_\pi. \quad (23)$$

This implies that $g_A \sim 1/\sqrt{n}$ if A and B are close neighbors (in mass), and falls off fast when A and B become distant neighbors. This is another argument in favor of the Nambu–Goldstone realization of the chiral symmetry. (The second expression in Eq. (23) is due to the generalized Goldberger–Treiman relation, see Appendix B.)

7 Holographic string/gauge duals

We will discuss two different approaches: top-down and bottom-up. In both the holographic description of hadrons with the fifth coordinate is used. In the first calculation by Sakai and Sugimoto [17] the gauge theory is represented by a stack of N_c D4-branes in ten-dimensional type IIA string theory. One of the D4 dimensions is compactified on the circle S^1 to break supersymmetry of the superstring theory by anti-periodic boundary conditions for fermions. The quarks are introduced by N_f test D8- $\overline{\text{D8}}$ pairs living in dimension orthogonal to S^1 . They are associated with strings connecting a D4 brane with D8 or $\overline{\text{D8}}$; the $U_L(N_f) \times U_R(N_f)$ chiral symmetry of QCD is realized as a gauge symmetry of the N_f D8- $\overline{\text{D8}}$ pairs.

In the limit of large N_c and large 't Hooft coupling the supergravity approximation to string theory is applied. The solution for the metric is characterized by existence of the horizon, $U > U_{\text{KK}}$, in the radial coordinate U transverse to the D4 branes. As $U \rightarrow U_{\text{KK}}$ the radius of S^1 shrinks to zero and D8/ $\overline{\text{D8}}$ branes merge to form a single component of the D8-branes, see Fig. 5. Only the diagonal $U(N_f)$ survives on the resulting D8 brane. This explicitly shows spontaneous breaking of the $U_L(N_f) \times U_R(N_f)$ chiral symmetry.

In application to the vector, axial-vector and spin-zero fields the above construction generates the action

$$S^{\text{SS}} = \kappa \int d^4x dz \text{Tr} \left[K^{-1/3} \frac{1}{2} F_{\mu\nu}^2 + K F_{\mu z}^2 \right] \quad (24)$$

where

$$K = 1 + z^2, \quad \kappa = \frac{\lambda N_c}{108\pi^3}. \quad (25)$$

This expression is written in five dimensions, z is the holographic coordinate. Boundaries at positive and negative z -asymptotics correspond to $V \pm A$ combinations of

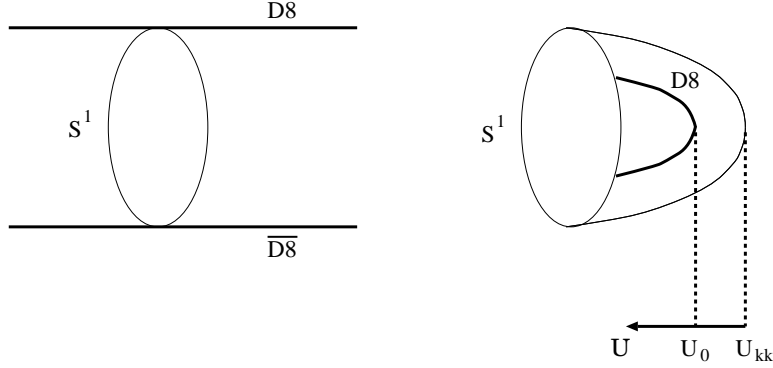


Figure 5: A sketch of D8 and $\overline{\text{D8}}$ branes from [17].

the gauge fields while A_z is associated with the pseudoscalar field. The conformal symmetry is softly broken; one-dimensional quantum mechanics in the holographic direction gives the mass spectrum. The massless pion is built-in and does not decouple from high excitations. Related to this is a nice feature of the construction: a Skymion nature of baryons. What is absent in this picture is the linearity of the Regge trajectories; they are parabolic instead.

In the second, bottom-up approach, the authors introduce independent fields in each channel, i.e. vector, axial-vector and spin-zero fields. They also fix the metric and dilaton field to get linear dependence of M^2 on n_r and L ,

$$S^{\text{KKSS}} = \kappa \int d^4x dz e^{\Phi(z)} \sqrt{g} \left[-|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2} (F_L^2 + F_R^2) \right] \quad (26)$$

where

$$\Phi = z^2, \quad g_5^2 = \frac{12\pi^2}{N_c}, \quad (27)$$

while the metric is given by

$$ds^2 = \frac{1}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2). \quad (28)$$

As in the previous case the quantum-mechanics eigenvalues along the holographic coordinate z give the meson spectrum M^2 in four dimensions. High excitations are characterized by the growing sizes both in 4d and in the holographic coordinate, $z \sim \sqrt{n}$.

The linear chiral symmetry implies that $\langle X \rangle = 0$. Then ρ and a_1 mesons are degenerate together with the tower of their excitations. The splitting between the

ρ and a_1 which certainly exists in nature is due to condensation of the X field containing pion and its scalar partner, $\langle X \rangle \neq 0$. In the KKSS approach this splitting is proportional to X^2/z^2 . Since the characteristic $z^2 \sim n$ for high radial excitations the mass splitting (and corresponding symmetry restoration) depends on behavior of X at large z . KKSS assumed that $X \rightarrow \text{const}$ which means that the X contribution diminishes with n implying asymptotic restoration of the chiral symmetry. Clearly it simultaneously introduces nonlinearity of the n dependence of M^2 , as illustrated in Fig. 6. The solid lines in this figure depict trajectories for $\langle X \rangle = 0$ while the dashed

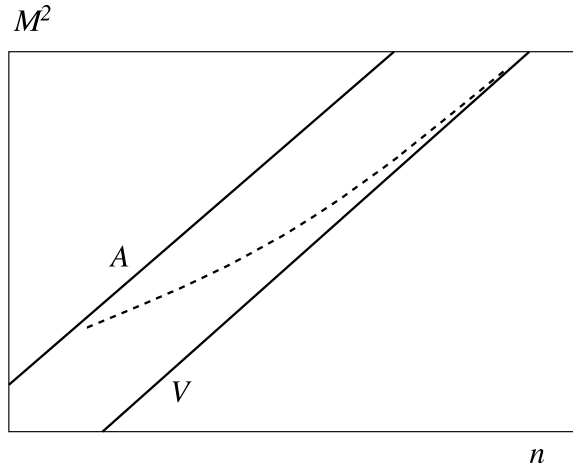


Figure 6: M^2 versus n for the vector and axial vector resonances from KKSS.

line represent the $1/n$ deformation of the axial masses due to $\langle X \rangle = \text{const}$. This does not seem to be compatible with phenomenology of known resonances which form linear trajectories starting from $n \sim 1$.

Moreover, there is no theoretical justification for the above assumption $X \rightarrow \text{const}$: despite the fact the dimension for the X field is (-3) there is no physical instability. In the recent papers [20,21] the authors argued that instead one can take $X \sim z$ at large z . Then the M^2 splittings between the chiral partners do not depend on n which fits well with the linearity and equidistance of the Regge trajectories.

8 Chiral symmetry in the Euclidean domain and the meson spectrum

Operator product expansion (OPE) shows that the chiral symmetry is restored at short Euclidean distances. For instance, for the difference of two-point functions

of scalar and pseudoscalar currents we have the following OPE expansion at large Euclidean momentum Q

$$\Pi_S(Q) - \Pi_P(Q) \sim \frac{g^2 \langle \bar{q}q \rangle^2}{Q^4} + \dots \quad (29)$$

as was shown long ago in [35]. Here

$$\begin{aligned} \Pi_{S,P}(Q) &= -i \int d^4x e^{iqx} T \langle j_{S,P}(x) j_{S,P}(0) \rangle, \\ j_S &= \bar{q}q, \quad j_P = i\bar{q}\gamma_5 q. \end{aligned} \quad (30)$$

We see that the $\Pi_S - \Pi_P$ difference is expressed in terms of the quark condensate $\langle \bar{q}q \rangle$ which is the order parameter for the chiral symmetry breaking. We observe a rapid chiral symmetry restoration at large Q .

The question we address is what constraints on the spectrum follow from this behavior. Of course, for any given spectral model Eq. (29) does lead to constraints. For instance, for equidistant spectra, $M_n^2 \propto n$, and equal residues one can rule out different slopes (see Ref. [20] for a noncomplying example).

However, the Euclidean behavior is sensitive strictly speaking only to averaged features of the spectral densities. For instance, one can replace the sum of the narrow peaks by a smooth curve (quark-antiquark continuum) introducing only an exponential correction of the type $\exp(-Q)$ not visible in OPE.⁸ Thus, there is no unique prediction for the mass splitting of the chiral partners, as was emphasized, in particular, in Refs. [25, 38].

Let us consider some limiting cases. The first scenario is given by an appropriate splitting of the lowest resonances in the scalar and pseudoscalar channels,

$$\Pi_S(Q) - \Pi_P(Q) = F^2 \left(\frac{1}{Q^2 + M_S^2} - \frac{1}{Q^2 + M_P^2} \right) \rightarrow -\frac{F^2(M_S^2 - M_P^2)}{Q^4}. \quad (31)$$

Here we assumed that the residues are the same to eliminate the unwanted $1/Q^2$ term. In addition, we assumed all higher resonances to be fully degenerate, both in masses and residues.

However, the full degeneracy for higher states can be replaced by a much weaker requirement of a local conspiracy: the scalar and pseudoscalar resonances can be split into local in energy groups such that their contributions into the dispersion integrals are maintained the same.

⁸ Let us remind that this circumstance was used by Migdal [36] long ago.

The simple illustration of how this might work is as follows. Consider a pair with some large M in Eq. (31). This pair contributes $F^2 \Delta M^2$ to the coefficient of $1/Q^4$. The residues grow with M as M^2 because in the continuum (quark) spectral densities $\text{Im}\Pi_{S,P}(s) \sim s$. This implies that $M^2 \Delta M^2 \sim \text{const}$ for high excitations. This estimate was obtained in Ref. [37].⁹ Clearly, the group of conspirators can be enlarged. If we included two scalars and two pseudoscalars in such a group we could arrange for cancellation of $1/Q^4$ terms without requiring $\Delta M^2 \rightarrow 0$. Involving more and more conspirators we could achieve exponential accuracy mentioned above [36].

Note that even if $\Delta M^2 \rightarrow 0$ at large M , it is not sufficient to conclude that this pair belongs to the same linear chiral multiplet.

9 Baryons

The problem of χ SR can be addressed in baryons, with reservations, only at intermediate values of J and n . This is due to the fact that baryon masses grow with N_c and their decay widths, generally speaking, do not vanish in the limit $N_c \rightarrow \infty$. We will discuss χ SR in baryons with due caution.

There are a few folklore statements regarding baryons which do not seem to fall in one and the same picture. Let us briefly review them.

It is believed that the nucleon and Δ Regge trajectories are linear up to $J = 9/2$, and even higher in certain instances [39]. It is firmly established that for the ground states, N and Δ , there are no degenerate parity partners in the spectrum. At the same time, Particle Data Group reports degenerate $I(J^P) = \frac{1}{2} \left(\frac{5}{2}^\pm \right)$ states on the leading trajectory. With less certainty one can speak of parity degeneracy on the leading trajectory at $J = 9/2$.

What kind of degeneracy should one expect if χ SR takes place? Instead of the quark-antiquark pairs in the meson case, for baryons we should consider quark triplets

$$q_L q_L q_L, q_L q_L q_R, q_L q_R q_R, q_R q_R q_R. \quad (32)$$

The $U(1)_A$ charges of these triplets are $\{3, 1, -1, -3\}$. For the total quark spin $S = 3/2$ we have two types of the chiral multiplets differing in their $U(1)_A$ charges: one with the $U(1)_A$ charge 3, and one with the $U(1)_A$ charge 1. For $S = 1/2$ there are three types: one with the $U(1)_A$ charge 3, and two with the $U(1)_A$ charge 1. As in mesons, long strings imply spin degeneracy, i.e. degeneracy between

⁹ Note that a similar estimate gives the correct mass splitting in the 2d 't Hooft model. In this case F^2 does not grow with M implying $\Delta M^2 \sim \text{const}$.

distinct $U(1)_A$ charges. Such high level of degeneracy is not yet observed in the baryon spectrum.

From the Regge theory side parity degeneracy in baryons follows from the Gribov–MacDowell symmetry and linearity of baryonic Regge trajectories [39–41]. The linearity of the nucleon trajectory is supported also by data at negative t , in the scattering region. The absence of the parity doublers for N and Δ and their apparent presence at $J = 5/2$ could be reconciled with the linearity of the $P = -1$ Regge trajectory if the residues of the lowest states vanish [23]. This is a rather contrived scenario, though. If we do not want such a contrived scenario then the absence of the Gribov–MacDowell parity doubling for N and Δ could signal that (some of) the baryon trajectories are nonlinear to a significant extent. Then a direct experimental confirmation of this nonlinearity is a must.

As was noted in [23] (see also [42] for an earlier discussion), the meson and baryon Regge trajectories are similar in many respects, in particular, the slopes are practically equal. This might mean that for high excitations, when the connecting string is long, the difference between mesons and baryons is only at the endpoints: quark and antiquark in the former case and quark and diquark in the latter [23, 24, 42].¹⁰ If this is the case, the emergence of the parity doubling at large J could be a natural consequence of the fact that “good” diquarks (i.e. those with the vanishing spin and isospin) can be both scalars (i.e. $P = +1$) and pseudoscalars (i.e. $P = -1$). For instance, in the instanton liquid model the scalar diquark structure is very similar to that of pions [44], while pseudoscalar diquarks are similar to σ . Due to the strongest attraction, the scalar diquarks are the lightest, with an effective mass around 200 MeV [24, 44]. In the pseudoscalar diquarks the attraction is weaker, but if they are indeed similar to σ their mass can be as low as ~ 400 MeV [45]. Since in high excitations the diquarks are ultrarelativistic, their mass m enters in the baryon mass in the form m^2/p_{char} , where p_{char} is a characteristic diquark momentum.

At small J , when there are no long strings, the quark-diquark configuration is not necessarily dominant over three-quark configurations. This would naturally explain a curvature in the $P = -1$ baryon trajectory at small J and the emergence of degeneracy with the $P = +1$ baryon trajectory at large J .

If this explanation is correct the corresponding parity degeneracy has nothing to do with χ SR. Indeed, in the case of the linear realization of the chiral symmetry, the $U(1)_A$ charge is conserved. Then, similar to the mesonic case, the opposite parity states in the chiral multiplets are due to the shift of the orbital momentum by one unit rather than due to the passage from the scalar diquark to pseudoscalar one or vice versa. Just as in the meson case, this situation implies incompatibility of the

¹⁰Additional arguments on diquarks can be found in [43].

χ SR with the semiclassical limit.

A remark is in order here concerning a recent analysis [10] of the pion coupling to (excited) nucleons. A significant suppression of the pion coupling was found in a number of decays of excited nucleons into $N\pi$ implying, through the generalized Goldberger–Treiman relation, a suppression in the corresponding axial couplings. We would like to point out that this observation is insufficient for the conclusion of χ SR (although, it is necessary, of course).

Indeed, in the linearly realized chiral symmetry all transitions inside chiral multiplets have $g_A = 1$ (up to Clebsch-Gordan coefficients), while in the transitions leading outside the given chiral multiplet $g_A = 0$. In the Nambu–Goldstone mode the values of g_A for all transitions can be arbitrary. In particular, in the pion decays of highly excited states (long strings) it is natural to expect that g_A ’s are suppressed, see Eq. (23) and the subsequent discussion. To demonstrate that χ SR takes place one needs to identify degenerate chiral partners and demonstrate that $g_A = 1$ for transitions inside the chiral multiplet.

10 Conclusions

This article grew as a continuation of the ongoing heated debate in the literature regarding asymptotic symmetries of the meson spectrum in QCD [2, 5–8, 25, 33, 37, 46, 47], and numerous discussions of this issue at various conferences. At an early stage we believed that χ SR could be natural in QCD. Further more careful studies made us change our minds.

Our analysis consists of two parts: the first one is based on the existing Regge phenomenology; in the second part we try to combine various theoretical arguments, such as quasiclassical considerations, AdS/QCD, and so on. All arguments are consistent with the absence of the chiral symmetry restoration in the *observed* meson spectrum.

Given the abundant Regge phenomenology plus plausible theoretical arguments, we believe that the chiral symmetry is not restored in highly excited mesons. Various arguments show that ΔM^2 for would-be chiral partners does not depend on n . It means that $\Delta M \sim 1/M$ and decreases for high excitations as $1/\sqrt{n}$. This is insufficient for chiral symmetry restoration. The restoration takes place only when the chiral splitting is much smaller than the splitting between, say, the neighboring radial excitations. In actuality the values of ΔM_{\pm} in the would-be chiral multiplet are of the same order of magnitude as the splittings of the mesons lying on distinct daughter trajectories. There are also no obvious reasons for g_A to approach unity in the transitions between the chiral partners, moreover, we argued that it does not.

Our consideration is not a theorem but, rather, a physical argument. In the present-day theory it seems impossible to prove a theorem of no χ SR at large n, J , beyond the explored domain of resonances. If such χ SR does take place, the quasi-classical picture must badly fail, for reasons of which we have no clue. The Regge trajectories *must* curve in an “intermediate window” which, by itself, would present a very remarkable phenomenon.

The issue of the microscopic realization of the Gribov–MacDowell symmetry in baryons is contentious. Perhaps, the baryon data hint that linearity must be abandoned at least for some trajectories. For a breakthrough, a fully developed theory which would combine a picture of long strings for high excitations with the chiral symmetry of massless quarks at the endpoints is badly needed.

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Appendix A

In this Appendix we review some material relevant to the linear realization of the chiral symmetry.

If both left- and right-handed $SU(N_f)$'s are linearly realized, hadronic states must fall into degenerate multiplets of the full chiral symmetry. The degeneracy is lifted by an $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$ spontaneous breaking of the symmetry. Let us first briefly review an appropriate representation theory both for mesonic and baryonic states. These representations were studied long ago, even before the advent of QCD [48]. We would not touch the case of heavy-light mesons which was also studied [49].

We will present the construction using interpolating meson and baryon currents written in terms of quark fields. While this is done for an illustrative purpose some features like operator twists could be related with the stringy nature of the excited states.

A.1 Chiral symmetry: linear realization for mesons

In terms of the quark fields the $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry is conveniently represented in terms left- and right-handed Weyl spinors, $q_{L,R} = (1 \mp \gamma_5)q/2$,

$$[q_L]_{\alpha}^{if}, \quad [q_R]_{\dot{\alpha}}^{i\bar{f}}, \quad (\text{A.1})$$

where $\alpha, \dot{\alpha} = 1, 2$ are spinorial indices of the Lorentz group, $i = 1, \dots, N_c$ is the color index and $f, \bar{f} = 1, \dots, N_f$ are subflavor indices of two independent, left and right, $SU(N_f)$. The chiral symmetry transformations are

$$q_L^f \rightarrow L_g^f q_L^g, \quad q_R^{\bar{f}} \rightarrow R_{\bar{g}}^{\bar{f}} q_R^{\bar{g}}, \quad (\text{A.2})$$

where L and R are the $SU(N_f)_{L,R}$ matrices. Classically QCD has also $U(1)_L \times U(1)_R$ invariance,

$$q_L \rightarrow e^{i\eta_L} q_L, \quad q_R \rightarrow e^{i\eta_R} q_R, \quad (\text{A.3})$$

The diagonal $U(1)_V$ (when $\eta_L = \eta_R = \eta_B/N_c$) is associated with the baryon charge while the axial $U(1)_A$ (when $\eta_L = -\eta_R = \eta_A$) becomes anomalous at the quantum level. This dynamical breaking of $U(1)_A$ is suppressed at large N_c together with quark-gluon mixing. It also dies out for high excitations corresponding to short distances so we deal there with the asymptotic $U(N_f)_L \times U(N_f)_R$ symmetry.

The interpolating fields for colorless hadrons can be simply constructed from quark fields. Let us consider spin zero mesons. They are described by the matrix M ,

$$M_{\bar{f}}^f = [\bar{q}_R]_{i\bar{f}}^\alpha [q_L]_\alpha^{if} = \bar{q}_{\bar{f}} \frac{1 - \gamma_5}{2} q^f. \quad (\text{A.4})$$

The baryon charge of M clearly vanishes while the $U(1)_A$ charge is 2. The meson matrix M realizes the $\{N_f, N_f\}$ representation of $SU(N_f)_L \times SU(N_f)_R$ and contains $2N_f^2$ real fields. The reflection of space coordinates, P , which transforms q_L^{if} to $q_R^{i\bar{f}}$ and vice versa, acts on the matrix $M_{\bar{f}}^f$ as

$$PM = M^\dagger. \quad (\text{A.5})$$

It means that the Hermitian part of M describes N_f^2 scalars while anti-Hermitian part represents N_f^2 pseudoscalars. In terms of the diagonal $SU(N_f)_V$ symmetry (when $L = R$) these N_f^2 fields form the adjoint representation and the singlet.

This construction can be easily generalized to include higher spins, the Regge recurrences of spin zero,

$$[M_{\mu_1 \dots \mu_n}]_{\bar{f}}^f = [\bar{q}_R]_{i\bar{f}}^\alpha \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_n} [q_L]_\alpha^{if}. \quad (\text{A.6})$$

All these operators have leading twist 3.

Starting from spin 1 there exist interpolating $q\bar{q}$ operators of a different chiral structure. In case of spin 1 mesons one can introduce

$$\left[V_\mu^L \right]_g^f = \sigma_\mu^{\alpha\dot{\alpha}} [\bar{q}_L]_{\dot{\alpha}ig} [q_L]_\alpha^{if} = \bar{q}_g \gamma_\mu \frac{1 - \gamma_5}{2} q^f, \quad (\text{A.7})$$

where $\sigma^\mu = \{1, \vec{\sigma}\}$. The corresponding baryon and $U(1)_A$ charges are zero. Subtracting the trace we get the $\{N_f^2 - 1, 1\}$ representation while the trace part is the $\{1, 1\}$ representation of $SU(N_f)_L \times SU(N_f)_R$. The matrix V_μ^L is Hermitian so it represent N_f^2 fields of spin 1. These fields are singlets of $SU(N_f)_R$ and adjoints or singlets of $SU(N_f)_L$ (as well as $SU(N_f)_V$). Under the parity transformation V_μ^L goes to

$$\left[V_\mu^R \right]_{\bar{g}}^{\bar{f}} = \sigma_\mu^{\alpha\dot{\alpha}} [\bar{q}_R]_{\alpha i\bar{g}} [q_R]_{\dot{\alpha}}^{i\bar{f}} = \bar{q}_{\bar{g}} \gamma_\mu \frac{1 + \gamma_5}{2} q^{\bar{f}}. \quad (\text{A.8})$$

The vector and axial-vector particles are described by a sum and difference of V_μ^L and V_μ^R . The Regge recurrences are obtained in the same as in Eq. (A.6). All these operators have twist 2.

As was discussed in the literature, spin 1 mesons can be also described by antisymmetric tensor field which is the $(0, 1) + (1, 0)$ representation of Lorentz group instead of $(1/2, 1/2)$ used above,

$$\left[H_{\mu\nu}\right]_{\bar{f}}^f = [\sigma_\mu \bar{\sigma}_\nu]^{\alpha\beta} \left\{ [\bar{q}_R]_{\alpha i \bar{f}} [q_L]_{\beta}^{if} + \alpha \leftrightarrow \beta \right\} = \bar{q}_{\bar{f}} \sigma_{\mu\nu} \frac{1 - \gamma_5}{2} q^f, \quad (\text{A.9})$$

where $\bar{\sigma}^\mu = \{1, -\vec{\sigma}\}$. The chiral features of this tensor current are different from $[V_\mu^L]_g^f$ but the same as those of spin 0 fields $M_{\bar{f}}^f$, Eq. (A.4), and their Regge recurrences $[M_{\mu_1 \dots \mu_n}]_{\bar{f}}^f$, Eq. (A.6). Moreover, by applying the total derivative we see that the tensor current $[H_{\mu\nu}]_{\bar{f}}^f$ is equivalent to $[M_\mu]_{\bar{f}}^f$. Indeed,

$$\partial^\nu [H_{\mu\nu}]_{\bar{f}}^f = -i \bar{q}_{\bar{f}} \overleftrightarrow{D}_\mu \frac{1 - \gamma_5}{2} q^f. \quad (\text{A.10})$$

Thus, for any given spin we have two types of the chiral multiplets: charged and neutral with respect to $U(1)_A$. Each multiplet contains $2N_f^2$ states. This accounts for degeneracy of flavor adjoints and singlets in the large N_c limit in case of the $U(1)_A$ neutral interpolating currents, as in Eqs. (A.7, A.8). Each multiplet contains N_f^2 states of each parity. For the $U(1)_A$ neutral multiplets $CP = 1$ (for electrically neutral states) while in the $U(1)_A$ charged multiplets $CP = -1$.

Spin zero is special: only higher twist $U(1)_A$ neutral operators of the type $\bar{q}_g \gamma^\mu (1 - \gamma_5) G_{\mu\nu} D^\nu q^f$ are possible. These operators correspond to hybrid mesons rather than to quark-antiquark ones.

We pause here to make two remarks about the $N_f = 2$ case. Due to quasireality of the fundamental representation of $SU(2)$ the eight-dimensional representation of $SU(2)_L \times SU(2)_R$ given by 2×2 matrix $M_{\bar{f}}^f$ becomes reducible and can be split into two four-dimensional ones. This can be done by imposing the group-invariant conditions,

$$\tau_2 M_\pm^* \tau_2 = \pm M_\pm. \quad (\text{A.11})$$

Then

$$M_+ = \sigma - i \boldsymbol{\tau} \boldsymbol{\pi}, \quad M_- = i\eta + \boldsymbol{\tau} \boldsymbol{\sigma}, \quad (\text{A.12})$$

where all fields are real. The quadruplet M_+ contains the isosinglet scalar σ and the isotriplet of pseudoscalars $\boldsymbol{\pi}$ while in M_- the pseudoscalar η is isosinglet and scalars form the isotriplet $\boldsymbol{\sigma}$.

However, as we mentioned above the asymptotic symmetry includes also $U(1)_A$ (the vector $U(1)_B$ does not act on mesons). The $U(1)_A$ transformations mix M_+ and M_- thus restoring eight-dimensional representation.

The second remark refers to a hypothetical QCD-like theory (which may or may not be useful, say, in technicolor) rather than to actual QCD. Assume that we consider an $SU(N_c)$ Yang–Mills theory with two Dirac quarks in the adjoint representation of $SU(N_c)$, or, which is the same, four Weyl spinors in the adjoint. In this case, as well-known [50], the pattern of the χ SB is different from that in conventional QCD, namely, $SU(4) \rightarrow O(4)$. Since the $O(4)$ symmetry which is isomorphic to $SU(2) \times SU(2)$ is strictly unbroken, all hadrons in this theory, including pions and other low-lying states, will be classified in multiplets of the exact $SU(2) \times SU(2)$ symmetry.

A.2 Chiral symmetry: linear realization for baryons

The baryon currents can be introduced in a similar way. They contains N_c quark fields so, for example, the baryon current with the maximal spin $N_c/2$ is

$$[B_{\alpha_1 \dots \alpha_{N_c}}^L]^{f_1 \dots f_{N_c}} = \epsilon_{i_1 \dots i_{N_c}} q_{L\alpha_1}^{i_1 f_1} \dots q_{L\alpha_{N_c}}^{i_{N_c} f_{N_c}}, \quad (\text{A.13})$$

where symmetrization over $\alpha_1 \dots \alpha_{N_c}$ as well as over $f_1 \dots f_{N_c}$ is implied. Because of symmetry in $f_1 \dots f_{N_c}$ the number of fields in the multiplet is equal to the binomial coefficient $C(N_c + N_f - 1, N_c)$. Their baryon charge is 1 and the $U(1)_A$ charge is N_c . The parity transformation relates $[B_{\alpha_1 \dots \alpha_{N_c}}^L]^{f_1 \dots f_{N_c}}$ to a similarly defined $[B_{\alpha_1 \dots \alpha_{N_c}}^R]^{\bar{f}_1 \dots \bar{f}_{N_c}}$.

This $B_L^{f_1 \dots f_{N_c}}, B_R^{\bar{f}_1 \dots \bar{f}_{N_c}}$ construction for the baryons does *not* allow one to introduce the chirally invariant mass, in contrast to the meson case. Indeed, the Lorentz invariant convolution $\bar{B}_{R\alpha_1 \dots \alpha_{N_c}}^{\bar{f}_1 \dots \bar{f}_{N_c}} B_{L\beta_1 \dots \beta_{N_c}}^{f_1 \dots f_{N_c}} \epsilon^{\alpha_1 \beta_1} \dots \epsilon^{\alpha_{N_c} \beta_{N_c}}$ contains no chiral singlet. To allow for the invariant mass one needs to add $\tilde{B}_L^{\bar{f}_1 \dots \bar{f}_{N_c}}, \tilde{B}_R^{f_1 \dots f_{N_c}}$: this is called mirror doubling [48]. These mirror baryons cannot be introduced just as a simple product of quark fields for which handedness of their Lorentz index defines handedness of of their chiral representation. One has to use objects such as covariant derivatives $D_{\alpha\dot{\alpha}} = (\sigma^\mu)_{\alpha\dot{\alpha}} D_\mu$ for the construction. This considerably increases the twist, while $t = N_c$ for $B_{\alpha_1 \dots \alpha_{N_c}}^L$ it twice as large, $t = 2N_c$, for the mirror current. This high twist could be an extra argument against asymptotic linear symmetry.

Note that the chiral invariant baryon mass requires mirror doubling for other spin and flavor assignments as well.

A.3 Hadronic couplings: $U(2)_L \times U(2)_R$ without spontaneous breaking

Let us consider couplings of spin 0 and 1 mesons to spin 1/2 baryons in the linearly realized $U(2)_L \times U(2)_R$ as an illustrative example. As we discussed above eight spin 0 mesons (σ , $\boldsymbol{\sigma}$, η , $\boldsymbol{\pi}$) are described by the matrix $M_{\bar{f}}^f$, the spin 1 isotriplet mesons ($\boldsymbol{\rho}$, \boldsymbol{a}_1) are given by the traceless $[V_\mu^L]_g^f$ and $[V_\mu^R]_{\bar{g}}^{\bar{f}}$ and baryons are presented by B_α^{Lf} , $B_{\dot{\alpha}}^{R\bar{f}}$ and \tilde{B}_α^{Lf} , $\tilde{B}_{\dot{\alpha}}^{R\bar{f}}$.

Free baryons are described by Lagrangian

$$\begin{aligned} \mathcal{L}_B = & i\bar{B}_{Lf}\gamma^\mu\partial_\mu B_L^f + i\bar{B}_{R\bar{f}}\gamma^\mu\partial_\mu B_R^{\bar{f}} + i\tilde{B}_{L\bar{f}}\gamma^\mu\partial_\mu \tilde{B}_L^{\bar{f}} + i\tilde{B}_{Rf}\gamma^\mu\partial_\mu \tilde{B}_R^f \\ & - m_B \left[\tilde{B}_{Rf} B_L^f + \bar{B}_{R\bar{f}} \tilde{B}_L^{\bar{f}} + \text{h.c.} \right] \end{aligned} \quad (\text{A.14})$$

It shows that the 4-component Dirac spinors are formed by $\{B_L^f, \tilde{B}_R^f\}$ and $\{\tilde{B}_L^{\bar{f}}, B_R^{\bar{f}}\}$. The parity conservation is reflected in symmetry under permutations: $B_L^f \leftrightarrow B_R^f$, $\tilde{B}_L^{\bar{f}} \leftrightarrow \tilde{B}_R^{\bar{f}}$. Thus, $1/2^+$ and $1/2^-$ 4-component spinors are

$$B_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} B_L + \tilde{B}_L \\ \tilde{B}_R + B_R \end{pmatrix}, \quad B_- = \frac{1}{\sqrt{2}} \begin{pmatrix} \tilde{B}_L - B_L \\ B_R - \tilde{B}_R \end{pmatrix}. \quad (\text{A.15})$$

In terms of B_\pm

$$\mathcal{L}_B = i\bar{B}_+\gamma^\mu\partial_\mu B_+ + i\bar{B}_-\gamma^\mu\partial_\mu B_- - m_B \bar{B}_+ B_+ - m_B \bar{B}_- B_-, \quad (\text{A.16})$$

where summation over flavors is implied.

Couplings of spin 0 mesons to baryons linear in the meson matrix $M_{\bar{f}}^f$ are

$$M_{\bar{f}}^f \left[h \bar{B}_{Lf} B_R^{\bar{f}} + \tilde{h} \bar{B}_{R\bar{f}} \tilde{B}_L^{\bar{f}} \right] + \text{h.c.}, \quad (\text{A.17})$$

where two constants h and \tilde{h} are real to maintain P -invariance. It implies that couplings, e.g., to pions written in terms of B_\pm are

$$-i \frac{h + \tilde{h}}{2} (\bar{B}_+ \boldsymbol{\tau} \boldsymbol{\pi} B_- - \bar{B}_- \boldsymbol{\tau} \boldsymbol{\pi} B_+) - i \frac{h - \tilde{h}}{2} (\bar{B}_+ \boldsymbol{\tau} \boldsymbol{\pi} \gamma_5 B_+ - \bar{B}_- \boldsymbol{\tau} \boldsymbol{\pi} \gamma_5 B_-). \quad (\text{A.18})$$

Cross-couplings linear in both B and \tilde{B} contain two meson fields, e.g.,

$$M_{\bar{f}}^f \bar{M}_g^{\bar{f}} \bar{B}_{Lf} \tilde{B}_R^g, \quad M_{\bar{f}}^f \partial_\mu \bar{M}_g^{\bar{f}} \bar{B}_{Rf} \gamma^\mu \tilde{B}_R^g. \quad (\text{A.19})$$

For spin 1 mesons described by $V_{\mu g}^{Lf}$, $V_{\mu g}^{R\bar{f}}$ couplings are

$$V_{\mu g}^{Lf} \left[g \bar{B}_{Lf} \gamma^\mu B_L^g + \tilde{g} \bar{\tilde{B}}_{Rf} \gamma^\mu \tilde{B}_R^g \right] + V_{\mu g}^{R\bar{f}} \left[g \bar{B}_{R\bar{f}} \gamma^\mu B_R^{\bar{g}} + \tilde{g} \bar{\tilde{B}}_{L\bar{f}} \gamma^\mu \tilde{B}_L^{\bar{g}} \right] + \text{h.c.} \quad (\text{A.20})$$

In particular for ρ meson it gives

$$\frac{g + \tilde{g}}{2} (\bar{B}_+ \boldsymbol{\tau} \boldsymbol{\rho}_\mu \gamma^\mu B_+ + \bar{B}_- \boldsymbol{\tau} \boldsymbol{\rho}_\mu \gamma^\mu B_-) - \frac{g - \tilde{g}}{2} (\bar{B}_+ \boldsymbol{\tau} \boldsymbol{\rho}_\mu \gamma^\mu \gamma_5 B_+ + \bar{B}_- \boldsymbol{\tau} \boldsymbol{\rho}_\mu \gamma^\mu \gamma_5 B_-). \quad (\text{A.21})$$

Again cross-coupling containing B and \tilde{B} are quadratic in meson matrices.

Appendix B: Spontaneous symmetry breaking and generalized Goldberger–Treiman relation

Spontaneous symmetry breaking can be introduced as nonvanishing vacuum average of the meson matrix $M_{\bar{f}}^f$. As we discussed above in the $N_f = 2$ case this matrix contains 8 real fields. There are two $\text{U}(2)_L \times \text{U}(2)_R$ invariants for this matrix, $\text{Tr} M M^\dagger$ and $|\text{Det} M|^2$. Correspondingly a generic matrix M can be presented in the form

$$M = \sigma e^{i\eta} U V, \quad (\text{B.1})$$

where σ and η are real numbers, the matrix $U \in \text{SU}(2)$, i.e. unitary and unimodular, like $U = \exp(-i\boldsymbol{\tau}\boldsymbol{\pi}/\sigma_0)$, and the matrix V is Hermitian and unimodular, i.e. $V = \exp(\boldsymbol{\tau}\boldsymbol{\sigma}/\sigma_0)$.

If $\langle V \rangle_0 \neq \mathbb{I}$ (i.e. $\langle \sigma_3 \rangle_0 \neq 0$) then $\text{U}(2)_L \times \text{U}(2)_R$ symmetry is spontaneously broken to $\text{U}(1)_B \times \text{U}(1)_{I_3}$ and six Goldstone bosons appear: triplet of pseudoscalars, $\boldsymbol{\pi}$, singlet pseudoscalar, η , and two scalars, $\sigma^\pm = (\sigma_1 \mp i\sigma_2)/\sqrt{2}$. Such a pattern of spontaneous breaking when the vector $\text{SU}(2)_V$ is broken is not allowed in QCD [51], it means that we should take $\langle V \rangle_0 = \mathbb{I}$.

Then the pattern of breaking is standard, $\text{U}(2)_L \times \text{U}(2)_R \rightarrow \text{U}(1)_B \times \text{SU}(2)_V$, with four Goldstones, $\boldsymbol{\pi}$ and η . The fourth one, η , is actually pseudo-Goldstone because the associated $\text{U}(1)_A$ symmetry is anomalous in QCD. Still we have to keep η , together with scalars σ and $\boldsymbol{\sigma}$, as massive partners of pions when considering asymptotically linear realization of the chiral symmetry.

To see how the spontaneous breaking acts on baryons let us substitute M in Eq. (A.17) by its vacuum average $\langle M \rangle_0 = \sigma_0 \mathbb{I}$. This lifts degeneracy in masses of $1/2^+$ and $1/2^-$ baryons,

$$m_{B_-} - m_{B_+} = \sigma_0 (h + \tilde{h}). \quad (\text{B.2})$$

Equation (A.18) shows that the same combination $h + \tilde{h}$ enters the $\pi B_- B_+$ coupling,

$$g_{\pi B_- B_+} = -i \frac{h + \tilde{h}}{2}, \quad (\text{B.3})$$

so

$$m_{B_-} - m_{B_+} = 2i\sigma_0 g_{\pi B_- B_+}. \quad (\text{B.4})$$

This equation is an analogue of the Goldberger–Treiman relation.

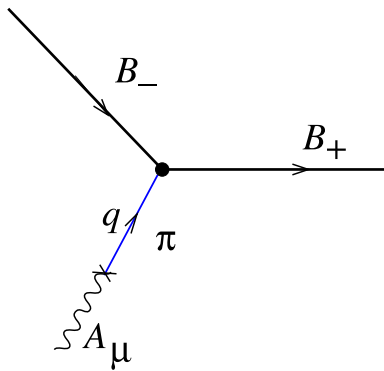


Figure 7: The pion pole in the B_- to B_+ matrix element of the axial current

Indeed, let us sandwich the axial current A_μ^3 between upper isospin components of B_\pm ,

$$\langle B_+ | 2A_\mu^3 | B_- \rangle = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \bar{B}_+ \gamma^\nu B_- , \quad (\text{B.5})$$

see Fig. 7 for notation. The expression in parentheses makes the axial current matrix element explicitly transverse, as is required by the axial current conservation in the chiral limit. Here we dropped an overall constant in front of $\bar{B}_+ \gamma^\mu B_-$ (an analogue of g_A) as it is expected to be 1 in the limit of heavy B_\pm .¹¹

Comparing the residue of the pole from Eq. (B.5) and Fig. 7 we immediately conclude that

$$2ig_{\pi B_- B_+} F_\pi = m_{B_-} - m_{B_+}. \quad (\text{B.6})$$

This is the same relation as Eq. (B.4) with the identification $\sigma_0 = F_\pi$. Equation (B.6) implies that for the n -th “radial” excitation $(\Delta m_\pm)_n$ scales as $(g_{\pi B_- B_+})_n$, and falls off with n with the same rate.

¹¹ Deviations from 1 are reflected in additions to $\pi B_- B_+$ vertex due to the couplings (A.19). The second of these couplings containing derivatives contributes after symmetry breaking.

We can apply a similar consideration to coupling of the axial-vector boson a_1 to B_-B_+ , its contribution to the nonpole part of the matrix element (B.5) is given by the same Fig. 7 with the substitution of π by a_1 . In this way we arrive at

$$\frac{2F_{a_1} g_{a_1 B_- B_+}}{m_{a_1}^2} = -1, \quad (\text{B.7})$$

where F_{a_1} is the coupling of a_1 to the axial current and m_{a_1} is its mass. The relation (B.7) demonstrates clearly that $g_{a_1 B_- B_+}$ does not decrease for high excitations in contrast with $g_{\pi B_- B_+}$.

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